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Dielectrophoretic Filtration in Time-Dependent Fields*

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Abstract

Theoretical analysis and experiments are presented on filtration of liquids by means of the dielectrophoretic effect in pulsating and ac fields. Quite satisfactory results are obtained in the ac case at frequencies above 4 kHz, even for a relatively conducting medium. It is also shown that as the force is time-dependent, allowance must be made for the interaction of the particles with the trapping surfaces.

In recent papers (*1, 2*) we extensively studied the possibility of liquid filtration by means of the dielectrophoretic effect, induced by applying a nonhomogeneous field to the medium to be filtered. These studies, while demonstrating the overall advantages of the method, were confined to dc fields in which the particle-trapping process is impeded as the conductivity of the medium increases. The study reported here showed that this difficulty can be overcome by recourse to time-dependent pulsating or ac fields (especially the latter). In addition, the time-dependent regime brought out phenomena which could not be followed under its dc counterpart, e.g., the interaction of the particles with the trapping surfaces.

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THEORETICAL CONSIDERATIONS

When a dc field E is suddenly applied at $t = 0$, the dielectrophoretic force F (*3*) on a sphere of radius R varies with time t:

$$F = 2\pi R^3 \varepsilon_1 \left[\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + 2\varepsilon_1} + 3 \frac{(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)}{(\varepsilon_2 + 2\varepsilon_1)(\sigma_2 + 2\sigma_1)} (1 - e^{-t/\tau}) \right] \nabla E^2 \tag{1}$$

where ε_1, σ_1 (ε_2, σ_2) are the dielectric constant and conductivity of the medium (particle), respectively, and τ is a characteristic time (relaxation time) given by $\tau = (\varepsilon_2 + 2\varepsilon_1)/(\sigma_2 + 2\sigma_1)$. As previously mentioned (*4*), two regimes can be distinguished: the dielectric regime when $t \ll \tau$ or σ_1, $\sigma_2 \to 0$, the force being

$$F_{\text{diel}} = 2\pi R^3 \varepsilon_1 \left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + 2\varepsilon_1} \right) \nabla E^2 \tag{2}$$

and the conduction regime when $t \gg \tau$, the force being

$$F_{\text{cond}} = 2\pi R^3 \varepsilon_1 \left(\frac{\sigma_2 - \sigma_1}{\sigma_2 + 2\sigma_1} \right) \nabla E^2 \tag{3}$$

For an alternating field $E(r,t) = E(r) \cos \omega t$, the force* is given by (*3*)

$$F = 2\pi R^3 \varepsilon_1 \left\{ \left[\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + 2\varepsilon_1} + 3 \frac{(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)}{\tau(\sigma_2 + 2\sigma_1)^2(1 + \omega^2\tau^2)} \right] \cos^2 \omega t + 3 \frac{(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)\omega}{(\sigma_2 + 2\sigma_1)^2(1 + \omega^2\tau^2)} \cos \omega t \sin \omega t \right\} \nabla E^2 \tag{4}$$

For the dielectric regime ($\omega\tau \gg 1$ or σ_1, $\sigma_2 \to 0$), we have

$$F_{\text{diel}} = 2\pi R^3 \varepsilon_1 \left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + 2\varepsilon_1} \right) \cos^2 \omega t \, \nabla E^2 \tag{5}$$

*The term proportional to $e^{-t/\tau}$, which describes the transient behavior of the force, was shown (3) to be significant only for $\omega\tau \sim 1$, while we are concerned with the extreme cases $\gg 1$ and $\ll 1$ representing the two regimes; accordingly, it is omitted in Eqs. (4)-(6).

and for the conduction regime ($\omega\tau \ll 1$)

$$F_{\text{cond}} = 2\pi R^3 \varepsilon_1 \left(\frac{\sigma_2 - \sigma_1}{\sigma_2 + 2\sigma_1} \right) \cos^2 \omega t \, \nabla E^2 \tag{6}$$

In our experiments the liquid medium was filtered by letting it flow between a pair of concentric electrodes in a space filled with spherical beads 2–6 mm in diameter, onto which the solid particles in the medium are trapped under the action of a dielectric force perpendicular to the flow direction. Besides their role as trapping foci, the beads distort the field lines and increase the local field gradient, which is a function of the distance from the apparatus axis and of the position of the chosen point relative to the beads. When the dielectrophoretic force is positive ($F > 0$), the particles are attracted toward the beads and trapped; when it is negative ($F < 0$), they are repelled and pass between the beads untrapped.

The description below is based on a similar hypothesis as in Ref. 5; namely, that a particle is trapped if its lateral displacement $y(t)$ from its point of entry (at distance r from the apparatus axis) reaches a critical value y_1. The displacement is especially difficult to determine since we do not know the exact variation of ∇E^2 in the vicinity of the beads. For the sake of definiteness, we assume that $y(t)$ will always be smaller than r and that a particle feels a force constant in space. (This is not true very close to the apparatus axis, but the number of particles in such a situation is small.) In other words, the effective gradient ∇E^2 is an average of the real gradient over distances equal to the mean spacing between the beads. In Ref. 5 we showed that this effective gradient is proportional to $V^2 r^{-a}$, where V is the applied potential between the electrodes and a is an exponent equal to 6 ± 1.

In the absence of the electric field, the particles flow parallel to the axis (no mean lateral displacement), and we call t_c, the time for a particle to cross the apparatus, t_c = (length of the electrodes)/(particle velocity relative to the apparatus). Now, if a particle reaches the critical distance y_1 in a time smaller than t_c, it is trapped. We can say that the lateral displacement y_c at t_c must be larger than y_1

$$y_c = y(t_c \geqslant y_1$$

Since $y(t_c)$ is dependent of r through ∇E^2, the equality $y(t_c) = y_1$ defines a trapping radius r_t such that if $r \leqslant r_t$, a particle is trapped. Furthermore, we assume than the percentage of the trapped particles is proportional to r_t^2.

We thus have to calculate the displacement y through the equation of motion.

1.1. dc Field

The equation of the motion for a particle with mass m is

$$m\frac{d^2y}{dt^2} + k\frac{dy}{dt} = F(t) \tag{7}$$

where*

$$F(t) = K[f_\infty + (f_0 - f_\infty)e^{-t/\tau}] \tag{8}$$

with $K = 2\pi R^3\varepsilon_1\nabla E^2$, $f_0 = (\varepsilon_2 - \varepsilon_1)/(\varepsilon_2 + 2\varepsilon_1)$, and $f_\infty = (\sigma_2 - \sigma_1)/(\sigma_2 + 2\sigma_1)$. Equation (7) may be rewritten in the following form:

$$\frac{d^2\left(\frac{my}{K}\right)}{dt^2} + \frac{k}{m}\frac{d}{dt}\left(\frac{my}{K}\right) = f_\infty + (f_0 - f_\infty)e^{-t/\tau} \tag{9a}$$

or

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} = \beta + \gamma e^{-t/\tau} \tag{9b}$$

with $x = my/K$, $\beta = f_\infty$, $\gamma = (f_0 - f_\infty)$, and $\alpha = k/m$. Under the dielectric regime ($\tau \to \infty$), Eq. (9b) is replaced by

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} = f_0 \tag{10}$$

Thus, to recover the dielectric regime, it suffices to substitute in Eq. (9b) $\beta \to f_0$, $\gamma \to 0$. The solution of Eq. (9b), with the initial conditions $t = 0$, $x = 0$, $dx/dt = 0$, reads

*Equation (8) is obtained from Eq. (1) by simple manipulations.

$$x = \frac{\beta}{\alpha}t - \frac{1}{\alpha}\left(\frac{\beta}{\alpha} + \frac{\gamma\tau}{\alpha\tau - 1}\right)(1 - e^{-\alpha t}) + \frac{\gamma\tau^2}{(\alpha\tau - 1)}(1 - e^{t/\tau}) \quad (11)^*$$

If $a = \tau^{-1}$, the solution is

$$x = \frac{\beta}{\alpha}t + \frac{\gamma - \beta}{\alpha^2}(1 - e^{-\alpha t}) - \frac{\gamma t}{\alpha}e^{-\alpha t} \quad (12)$$

Under the dielectric regime, we have, from Eq. (11):

$$x = \frac{f_0}{\alpha}t - \frac{f_0}{\alpha^2}(1 - e^{-\alpha t}) \quad (13)$$

$$x \simeq \frac{f_0}{\alpha}t \qquad (t \gg \alpha^{-1}) \quad (14)$$

or

$$y = \frac{2\pi R^3 \varepsilon_1 f_0 \nabla E^2}{\alpha m}t$$

taking into account that $\alpha m = k = 6\pi\eta R$

$$y = \frac{R^2 \varepsilon_1 f_0 \nabla E^2}{3\eta}t \quad (15)$$

We see that the trapping is more effective if (a) R is large, (b) η is low, and (c) ∇E^2 is large, as was experimentally verified.

If now τ is small, but still larger than α^{-1}, the displacement x is

$$x = \frac{\beta}{\alpha}t + \frac{\gamma\tau}{\alpha}(1 - e^{-t/\tau}) \quad (16)$$

*In the context of Eqs. (11) and (12), the order of magnitude of the coefficient α is of interest. Noting that it equals $k/m = 6\pi\eta R/(4/3)\pi R^3\rho$, where η is the viscosity and ρ is the apparent density of the particle, we find for the typical values $\eta = 10^{-2}$ cgs, $R = 30$ μm, and $\rho = 3 - \alpha \simeq 1.5 \times 10^3$ s^{-1}. It is thus seen that the term $e^{-\alpha t}$ in the two equations becomes negligible after several milliseconds.

If $\beta > 0$, x increases monotonically with time and there is trapping. However, if $\beta < 0$, we may have a change in the sign of x. If $t \ll \tau$, Eq. (16) reduces to $x = (\beta + \gamma)t/\alpha = f_0 t/\alpha$, and for large t, $x = \beta t/\alpha$. If $\beta = f_\infty$ and $(\beta + \gamma) = f_0$ have different signs (as was the case in some of our experiments (*1*)), there is a change in direction of the particle trajectory.

Finally, it is of interest how the displacement $x(t)$ behaves at $t \ll \tau, \alpha^{-1}$. Expansion of Eq. (11) or (12) yields

$$x = \frac{\beta + \gamma}{2} t^2 = \frac{f_0}{2} t^2 \tag{17}$$

In this range of very short time, the displacement is independent of the conductivities. We shall use this result for the case of a pulsating field.

1.2. Pulsating Field

Now we suppose that the field varies periodically in time with frequency $f = 1/T$. Over the period T, the field is nonzero for the interval $0 \leqslant t < \theta$ (θ being the pulse width) and zero for $\theta \leqslant t \leqslant T$, i.e., the fraction of the period in which the force acts on the particle is θ/T. The displacement $x(t)$ is smaller than in the dc case and thus we expect a decrease of the yield with the ratio θ/T.

First we consider very small values of θ ($\theta \ll \tau, \alpha^{-1}$). For $0 \leqslant t < \theta$ the displacement is given by Eq. (17).

For $\theta \leqslant t \leqslant T$, the equation of the motion is

$$\frac{d^2x}{dt^2} + \alpha x = 0$$

subject to the condition that for $t = \theta, x = f_0\theta^2/2$, and $dx/dt = f_0\theta$. We thus have for this interval

$$x = \frac{f_0\theta^2}{2} + \frac{f_0\theta}{\alpha}(1 - e^{-\alpha(t-\theta)}) \tag{18}$$

The total displacement after one period is

$$x(T) = \frac{f_0\theta^2}{2} + \frac{f_0\theta}{\alpha}[1 - e^{-\alpha(T-\theta)}]$$

and after N periods ($t_N = NT$)

$$x(t_N) = \frac{f_0\theta^2}{2T} t_N + \frac{f_0\theta t_N}{\alpha T}[1 - e^{-\alpha(T-\theta)}] \tag{19a}$$

Since it was supposed that $\theta \ll \alpha^{-1}$, Eq. (19a) reduces to

$$x(t_N) = \frac{f_0}{\alpha}\left(\frac{\theta}{T}\right) t_N[1 - e^{-\alpha(T-\theta)}] \tag{19b}$$

Compared with the dc case, there are two important differences. First, the velocity of the particle (perpendicular to the flow direction) is reduced, as expected, because of the factor θ/T appearing in Eq. (19b) (this is the only difference under the dielectric regime ($\beta = f_0$)). Second, in the general case β reduces to f_∞ and, as mentioned, f_0 and f_∞ may have different signs ($f_0 > 0, f_\infty < 0$). In that case the velocity would be negative in a dc field (i.e., no trapping) and positive in a pulsating field. This is the theoretical basis for the use of pulsating fields for filtration of conducting liquids with due allowance for the interaction effect (see below).

For large values of θ, $x(t)$ in the interval $0 \leqslant t \leqslant \theta$ is given not by Eq. (17) but by either Eq. (13) or (16). Under the dielectric regime, for instance, we find that $x(t_N)$ is now given by a more complicated expression:

$$x(t_N) = \frac{f_0\theta t_N}{\alpha T} - \frac{f_0 t_N}{\alpha^2 T}(1 - e^{-\alpha\theta})(1 - e^{-\alpha\theta} + e^{-\alpha T}) \tag{20}$$

which shows that $x(t_N)$ is always smaller than in the dc case.

1.3. ac Field

The equation of the motion is again given by Eq. (7), but $F(t)$ is as per Eq. (4). We have

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} = A' \cos^2 \omega t + B' \sin \omega t \cos \omega t \tag{21}$$

where

$$A' = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + 2\varepsilon_1} + \frac{3(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)}{(\sigma_2 + 2\sigma_1)^2(1 + \omega^2\tau^2)\tau} \tag{22a}$$

$$B' = \frac{3(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)\omega}{(\sigma_2 + 2\sigma_1)^2(1 + \omega^2\tau^2)} \tag{22b}$$

Comparisons of the last two expressions with Eqs. (5) and (6) shows that for both regimes $B' \to 0$, but under the dielectric regime $A' \to (\varepsilon_2 - \varepsilon_1)/(\varepsilon_2 + 2\varepsilon_1) = f_0$ and under the conduction regime $A' \to (\sigma_2 - \sigma_1)/(\sigma_2 + 2\sigma_1) = f_\infty$.

Equation (21) is more conveniently rewritten in the following form:

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} = A + B\sin(2\omega t + \delta) \tag{23}$$

with

$$A = \frac{A'}{2}, \qquad B = \frac{1}{2}\sqrt{A'^2 + B'^2}, \qquad \sin\delta = \frac{A'}{\sqrt{A'^2 + B'^2}}, \qquad \cos\delta = \frac{B'}{\sqrt{A'^2 + B'^2}}$$

The solution of Eq. (23) with $x = 0$, $dx/dt = 0$ if $t = 0$ reads

$$x(t) = \frac{A}{\alpha}t + \frac{B}{2\omega\sqrt{\alpha^2 + 4\omega^2}}[\cos(\delta - \varphi) - \cos(2\omega t + \delta - \varphi)] + \left[\frac{A}{\alpha^2} + \frac{B\sin(\delta - \varphi)}{\alpha\sqrt{\alpha^2 + 4\omega^2}}\right](e^{-\alpha t} - 1) \tag{24}$$

with

$$\cos\varphi = \alpha/\sqrt{\alpha^2 + 4\omega^2}, \qquad \sin\varphi = 2\omega/\sqrt{\alpha^2 + 4\omega^2}$$

As above, the term $e^{-\alpha t}$ is negligible since α is large. (Retention of the transient contribution implies a term proportional to $e^{-t/\tau}\sin\omega t$, which has an average value of zero when integrated over times larger than $2\pi/\omega$.) Here the expression for the dielectric and conduction regimes are identical since for both $A = B$ and $\delta = \pi/2$ the difference is in the value of A ($A \to f_0/2$ and $A \to f_\infty/2$, respectively). The common expression for $x(t)$ is

$$x(t) = \frac{A}{\alpha} t + \frac{A}{2\omega\sqrt{\alpha^2 + 4\omega^2}} \left[\sin \varphi - \cos \left(2\omega t + \frac{\pi}{2} - \varphi\right) \right] - \left[\frac{A}{\alpha^2} + \frac{A \cos \varphi}{\alpha \sqrt{\alpha^2 + 4\omega^2}} \right] \tag{25}$$

If $\omega \ll \alpha$, we have $\varphi = 0$, hence

$$x(t) = \frac{A}{\alpha} t + \frac{A}{2\omega\alpha} \sin 2\omega t - \frac{A}{2\alpha\omega} \tag{26}$$

but if $\omega \gg \alpha$, $\varphi = \pi/2$ and

$$x(t) = \frac{A}{\alpha} t + \frac{A}{4\omega^2} [1 - \cos 2\omega t] - \frac{A}{\alpha^2} \tag{27}$$

However, $A/4\omega^2$ is much smaller than A/α^2, and Eq. (27) reduces to

$$x(t) = \frac{A}{\alpha} t - \frac{A}{\alpha^2} \tag{28}$$

If the dielectric regime is already reached when $\omega \gg \tau^{-1}, \alpha$, we have an interesting situation where $x(t)$ may differ widely in the dc and ac fields. For $f_0 > 0$ but $f_\infty < 0$, we saw above that in dc, dx/dt is positive only for a short time (of the order of τ) and negative for $t \gg \tau$. However, in the ac field, following Eq. (28), the velocity dx/dt is positive. Clearly, this will be true only for $t \gg \tau$, since the transient contribution was neglected in the preceding equations. In other words, for short τ, trapping can only be expected in the ac field.

2. EXPERIMENTAL

We recall briefly the experimental technique and refer the reader to Ref. *1.* for a detailed description. The basic apparatus is made of two concentric electrodes with diameters of 5 cm for the external electrode and 2 mm for the internal one. The electrode length is 20 cm when we used beads made of $BaTiO_3$ (diameter 2–3 mm) and 30 cm when we used

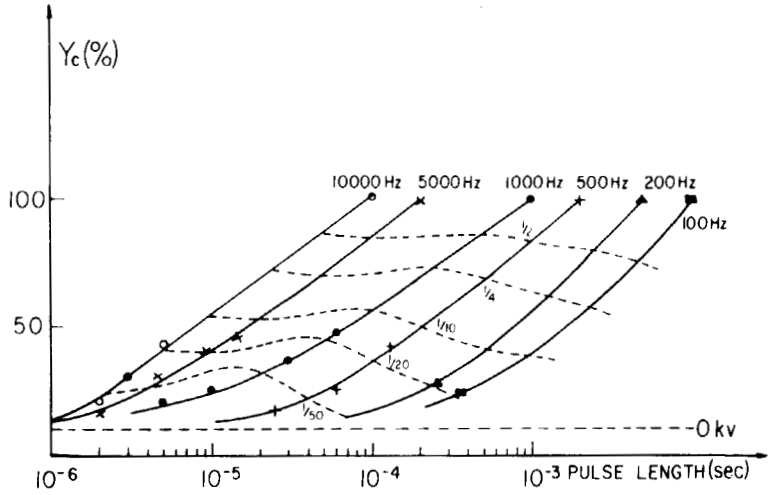

FIG. 1. Yield as a function of pulse width θ at different frequencies (including θ = T, representing the dc case). Dashed isopleths represent different θ/T ratios. Liquid, pure kerosene; particles, PVC.

glass beads (diameter 6 mm). The fluids in these experiments are pure kerosene and a mixture of kerosene with isopropanol in order to change the conductivity of the liquid by several orders of magnitude but with small variations in the dielectric constant. The flow rate is about 15 cm^3/min.

2.1. Pulsating Field

In the experiments with a pulsating field, the yield was measured as a function of the pulse width θ at different frequencies. Since the voltage of the pulse generator was of the order of 2–3 kV, we used beads made of $BaTiO_3$, which have a very high dielectric constant, thereby permitting increase of the field gradient and reduction of the voltage. The particles were PVC.

Figure 1 shows Y plotted against θ at different frequencies with pure kerosene as the medium. The pulse height was constant (2.5 kV) in all experiments. The results for the dc field (θ = T) are included for comparison. The yield is seen to decrease as θ decreases, the decrease

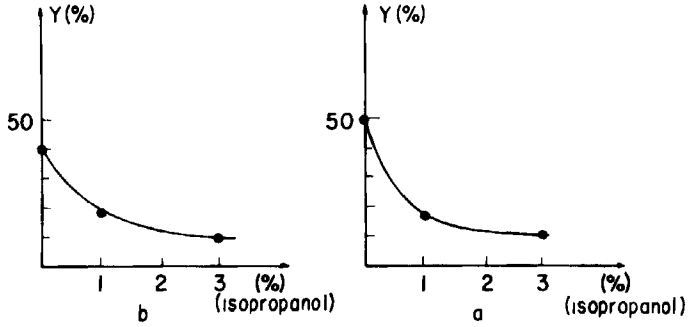

FIG. 2. Yield as a function of isopropanol percentage in kerosene in pulsating fields: (a) $f = 1$ kHz, $\theta = 60$ μs; (b) $f = 10$ kHz, $\theta = 8$ μs.

being more rapid at the lower frequencies* end ($f < 500$ Hz) and slower at the higher ones, as demonstrated by the dashed isopleths drawn for different θ/T ratios. All curves have maxima corresponding to about 2000 Hz.

The above results refer to pure kerosene for which, because of its high resistivity, the situation corresponds in practice to the dielectric regime. As mentioned earlier, there are situations with $f_0 > 0$ and $f_\infty < 0$ in which dc filtration is difficult when the dielectric constant of the particle is higher and its conductivity lower than those of the liquid. In order to realize such a situation, we increased the conductivity of the kerosene by adding isopropanol (see Ref. *1* for details) and measured the yield as a function of the isopropanol content. The results are given for two variants: $f = 1$ kHz, $\theta = 60$ μs (Fig. 2a); and $f = 10$ kHz, $\theta = 8$ μs (Fig. 2b). Both curves show that Y dropped to 10% (very close to the value for mechanical trapping) at 3% isopropanol, which is at variance with the theoretical prediction: at this isopropanol content τ is still relatively large (~ 0.1 s) and much larger than θ, so that the assumption ($\theta \ll \tau, \alpha^{-1}$) of the preceding section should apply. For the dc case the corresponding result was $Y = 68\%$, indicating a much larger loss of yield on transition from a dc to a pulsating field than in the case of pure kerosene. We return to this point in the next section.

*This decrease at lower frequencies implies that the time gap between the pulses suffices for a trapped particle to escape the bead to which it was being attracted; our estimate for the smallest gap required is 2×10^{-3} s.

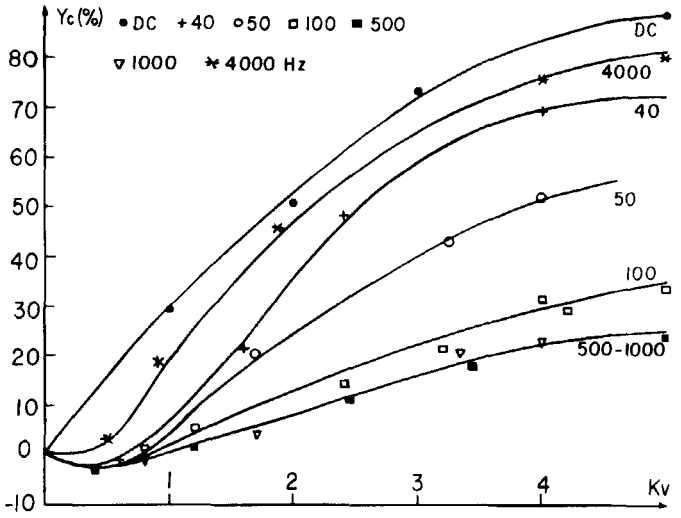

FIG. 3. Corrected yield Y_c versus applied voltage in an ac field at different frequencies (including the dc case). Liquid, pure kerosene (dielectric regime); particles, MgO.

2.2. ac Field

We present first the results of filtration of pure kerosene containing MgO particles in an apparatus filled with glass beads, as in Ref. *1*. Figure 3 shows the corrected yield Y_c as a function of the applied voltage at different frequencies, with the dc case ($f = 0$) again included for comparison. It is seen that at the beginning the yield decreases with increasing frequency but subsequently begins to increase. The curve for 4 kHz is very close to its dc counterpart. The same pattern is seen in Figs. 4 and 5, which present $Y_c(f)$ for a fixed voltage (4 kV) in a regular and a logarithmic scale, respectively. This behavior was completely unexpected, since the situation is that of the dielectric regime and whatever the value of ω, the mean value of x is given by $x(t) = (A/\alpha)t - A/\alpha^2$ (see Eqs. *26* and *28*).

Figure 5 also shows the result for a liquid with fairly high conductivity—70% kerosene, 30% isopropanol ($\sigma_1 \simeq 5 \times 10^{-7}\ \Omega^{-1}m^{-1}$). The yield is seen to be negative at low frequencies (as in the dc field) but begins to increase around 1 kHz and becomes positive at $f \geqslant 1200 \pm 100$ Hz.

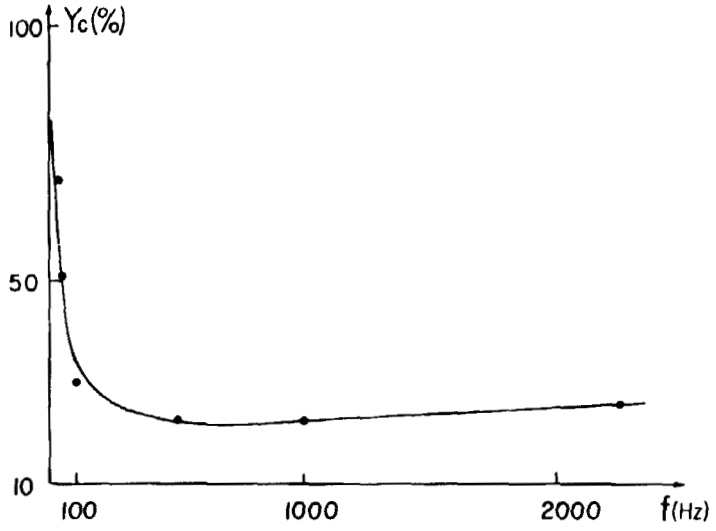

FIG. 4. Corrected yield versus frequency at $V = 4$ kV in pure kerosene (regular scale).

3. DISCUSSION

For the dielectric regime, the yield is readily obtainable from the results of the theoretical consideration of Section 1. If the voltage is constant (dc field), the trapping radius is given by

$$y(t_c) = \frac{R^2 \varepsilon_1 f_0 \nabla E^2}{3\eta} t_c = y_1 \tag{29}$$

Replacing ∇E^2 by the assumed expression $\nabla E^2 = BV^2 r^{-a}$, we have

$$\frac{R^2 \varepsilon_1 f_0 B V^2 r^{-a}}{3\eta} t_c = y_1 \tag{30a}$$

or

$$r_t = \left[\frac{y_1}{t_c} \frac{R^2 \varepsilon_1 f_0}{3\eta} BV^2 \right]^{1/a} \tag{30b}$$

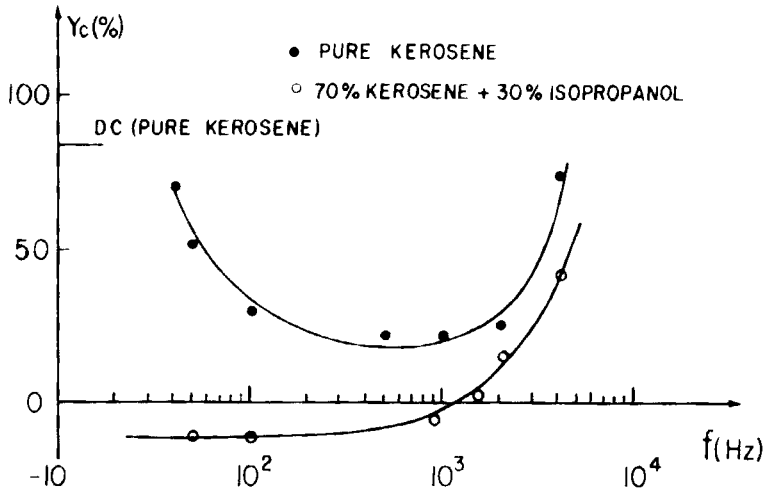

FIG. 5. Corrected yield versus frequency at $V = 4$ kV (logarithmic scale).

and we find

$$Y - Y_0 \propto r_t^2 = \left(\frac{y_1 B}{t_c}\right)^{2/a} \left(\frac{R^2 \varepsilon_1 f_0}{3\eta}\right)^{2/a} V^{4/a} \tag{31}$$

This expression is very similar to that used in Ref. 5, and, as mentioned above, it agrees well with the experimental results.

3.1. Pulsating Fields

At frequencies above 500 Hz and for $\theta/T < 0.5$, the assumption $\theta \leqslant \alpha^{-1}$ is valid and we can use Expression (19b). Recalling that $x = my/K$ with $K = 2\pi R^3 \varepsilon_1 \nabla E^2$ and $\alpha = 6\pi\eta R/m$, we have

$$y(t) = \frac{R^3 \varepsilon_1 f_0}{3\eta} \nabla E^2 t \, \frac{\theta}{T} (1 - e^{-\alpha(T-\theta)}) \tag{32}$$

$Y - Y_0$ is obtainable identically to the preceding case, through the determination of r_t. We find

$$Y - Y_0 = \left(\frac{y_1 B}{t_c}\right)^{2/a} \left(\frac{R^2 \varepsilon_1 f_0}{3\eta}\right)^{2/a} V^{4/a} \left[\frac{\theta}{T}(1 - e^{-\alpha(T-\theta)})\right]^{2/a} \tag{33}$$

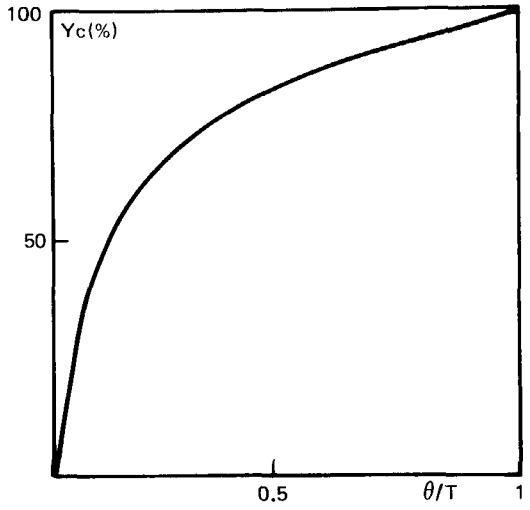

FIG. 6. Corrected yield as a function of θ/T in a pulsating field (regular scale). $f = 1$ kHz. Pure kerosene.

Figure 6 shows Y_c versus (θ/T). If a logarithmic scale is used (all other factors being constant), the result is a straight line with slope equal to $2/a$ (Fig. 7), as is indicated by Eq. (33) for $\alpha(T - \theta) \gg 1$. We find $2/a = 0.40$ and $a = 5$ (in agreement with the previous determinations); this implies a relatively high value of α, which is possible if the apparent density is low (as is the case for PVC) and the particles are small (~ 10 μm).

As mentioned in the preceding section, it had been expected that addition of the isopropanol would not affect the yield, and the only explanation we can offer for the observed decrease is modification of the mode of contact between solid and liquid, with considerable reduction of the minimum time gap ($\geqslant 2 \times 10^{-3}$ s) needed for a particle to escape the trapping bead.

3.2. ac Field

Here again we believe that the observed decrease in Y is due to escape of the particles. We propose to describe the motion of a particle near a bead by

$$\frac{d^2x}{dt^2} + \alpha \frac{dx}{dt} + \mu x = A(1 + \cos 2\omega t) \tag{34}$$

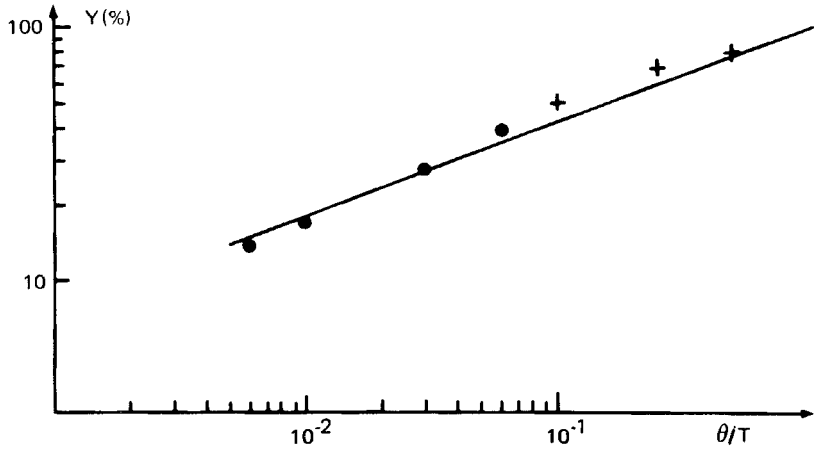

FIG. 7. Same as Fig. 6 (logarithmic scale): (●) measured, (+) extrapolated.

The additional term μx represents attraction of the particle by the bead. The solution of Eq. (34) is the superposition of a transient component

$$x_1(t) = e^{-\alpha t}[Me^{\sqrt{\alpha^2 - 4\mu}t} + Le^{-\sqrt{\alpha^2 - 4\mu}t}] \tag{35}$$

when L and M are two integration constants, and a steady-state component

$$x_2(t) = \frac{A}{\mu} + \frac{A}{\sqrt{(2\omega\alpha)^2 + (\mu - 4\omega^2)^2}} \sin(2\omega t + \lambda) \tag{36}$$

with

$$\cos\lambda = \frac{2\omega\alpha}{\sqrt{(2\omega\alpha)^2 + (\mu - 4\omega^2)^2}}$$

In Eq. (36) the constant term A/μ again represents the attractive effect, while the oscillating term proportional to $\sin(2\omega t + \lambda)$ represents the possibility of escape. Geometrically speaking, the motion of the particles has two components: $y(t) \propto x(t)$ perpendicular to the apparatus axis, and $z(t)$ parallel to the latter and due to liquid flow.

If the oscillating term is zero or negligible, there is no motion perpendicular to the axis (x = constant), indicating that the particle is trapped and $z(t)$ is also zero. By contrast, with the oscillating term

operative, the particle is not necessarily in contact with the bead surface and undergoes a $z(t)$ motion which provides the possibility of escape. Clearly, this effect is maximum at resonance, when the coefficient of sin $(2\omega t + \lambda)$ takes its largest value, i.e., for $2\omega_r = \sqrt{\mu}$. The corresponding value of $x(t)$ is

$$x = \frac{A}{4\omega_r^2} + \frac{A}{2\omega_r \alpha} \sin 2\omega t \tag{37}$$

Experimentally, we found $2\omega_r = 6300\ s^{-1}$ and estimate that α may be of the same order, namely 10^3-10^4. Thus, the condition for escape is for the constant term, and the amplitude* of the oscillating terms in Eq. (37) to be of the same order of magnitude, as observed.

We shall now analyze the results for the mixture kerosene + propanol in Fig. 5. The data for the liquid are $\varepsilon_1 = 2.75 \times 10^{-11}$ mks and $\sigma_1 = 5 \times 10^{-7}\ \Omega^{-1}m^{-1}$; those for the solid are $\varepsilon_2 = 9 \times 10^{-11}$ mks and $\sigma_2 \ll \sigma_1$. The relaxation time $\tau = (\varepsilon_2 + 2\varepsilon_1)/(\sigma_2 + 2\sigma_1)$ is short (1.5×10^{-4} s). We thus have need for the coefficients A' and B' of the dielectrophoretic force (see Eq. 22), and we get

$$A' = 0.43 - \frac{0.93}{1 + \omega^2\tau^2}, \qquad B' = -\frac{9.3 \times 10^{-5}\omega}{1 + \omega^2\tau^2}$$

We consider first the limit $\omega \ll \tau^{-1} = 10^4$ ($f \ll 1600$ Hz). From Eqs. (25) and (26) we find (since $\omega \ll 10^4\ s^{-1}$, we have also $\omega \ll \alpha$)

$$x = -\frac{0.47}{\alpha} t - \frac{0.47}{2\omega\alpha} \sin 2\omega t - \frac{0.47}{\alpha\omega} \tag{38}$$

The average value over several periods is thus negative; there is no trapping and Y_c is negative, as observed.

At the limit $\omega \gg \tau^{-1}$ (equivalent to $\omega \gg \alpha$), we find from Eq. (27)

$$x = \frac{0.21}{\alpha} t + \frac{0.21}{4\omega^2}(1 - \cos 2\omega t) - \frac{0.21}{\alpha} \tag{39}$$

The oscillating term in Eq. (39) has a very small amplitude and can be

*It should be noted that at high frequencies the amplitude tends to zero, and the oscillating term acts through the averages.

neglected. The average value of $x(t)$ is now positive and, as observed, there is trapping similar to that obtained in the dc case.

The yield is zero when A in Eq. (24) vanishes, i.e., when $A' = 0$ or

$$0.43(1 + \omega^2\tau^2) = 0.930$$

and we have $f = 1180$ Hz, which is quite close to the observed value (1200 Hz).

4. CONCLUSION

In this work we studied the process of filtration by means of the dielectrophoretic effect in time-dependent (pulsating and ac) fields at different frequencies. A theoretical analysis of the motion of the particle and experimental results were presented. On comparing the two approaches, we saw that two aspects of the trapping process have to be taken into account: first, the force and its dependence in time and space, and second, the interaction of the particle with the trapping surface, which is capable of considerable impairment of the efficiency of the process. In practical application the best results were obtained for the ac field at relatively high frequencies (> 4000 Hz), including successful filtration of a conducting liquid, which is unfeasible in a dc field. Use of ac fields in dielectrophoretic filtration is very promising, especially bearing in mind that high ac voltages in the 10-kHz range are easier to generate that dc ones.

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